Probabilistic Tracking of Fiber Pathways Using Dynamic Programming

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Introduction

Diffusion Tensor Imaging (DTI) has emerged as a powerful means of visualizing and tracking white matter fiber pathways in vivo. In DTI, a 3x3 diffusion tensor is obtained in which the eigenvectors and eigenvalues define an ellipsoid that describes the diffusive characteristics of water at each voxel in the image. Previous studies have shown that the average axonal orientation within each voxel can be derived and used to reconstruct 3D axonal tracts of interest [1]. The majority of these techniques, however, rely on propagation of the principal eigenvector to generate fiber paths, making them highly susceptible to noise and partial volume effects. Figure 1 illustrates how fiber paths that contain noise or branching (B and D) may cause propagation algorithms to terminate prematurely (A and C). To overcome these problems, we propose a statistical approach using dynamic programming to track curved trajectories through the volume.

Probability of N-length Path:

$$P\left[\pi_N(x_0, x_N)\right] = \prod_{i=0}^{N-1} \frac{e^{-\frac{1}{2}\left[v_i^T D^{-1}(x_i) v_i\right]}}{\left(2\pi\right)^3 \left[\det\left(D(x_i)\right)\right]^{\frac{1}{2}}} \qquad (1)$$

Cost Function:

$$\pi_N^*(x_0, x_N) = \underset{\pi \in P_x}{\operatorname{argmin}} \sum_{i=0}^{N-1} v_i^T D^{-1}(x_i) v_i + \ln\left(\lambda_1 \lambda_2 \lambda_3\right) \qquad (2)$$

Results

To compare our method with principal eigenvector tracking, we generated tracts of interest using the continuous tracking (FACT) method [3]. In Figure 2, fibers connecting the start and end regions (yellow) were reconstructed using dynamic programming (purple) and fibers passing through these regions were generated using FACT (blue). The cost per unit length between regions was also computed. The results of tracking along the anterior thalamic radiations via dynamic programming are in good agreement with those of FACT (A), but exhibit both a lower cost and cost per unit length (B). Dynamic programming was also employed to track to disjoint end regions along the anterior thalamic radiations and optic radiations, revealing the branching along these paths (C). The last row shows reconstructions of fibers in the corpus callosum that were not generated by the FACT method (D).

Below, we show how the normalized cost information (top right) can be used to obtain a connectivity strength metric for comparing tracts of interest. Once an optimal set of fibers has been computed, a subset of this can be obtained such that the worst fiber is within a specified percentage of the optimal fiber's probability. This probability threshold is a free parameter that is held constant for all tracts and the connectivity strength is then defined as the sum of the probability of all paths in this subset (bottom left). By computing this for each fiber tract, we show that regions joined by an abundance of high probability paths, such as the anterior thalamic radiations, will exhibit a strong connectivity strength, while those that progress into a region of relatively low anisotropy, such as the second branch of the optic radiations, will show a lower connectivity strength.

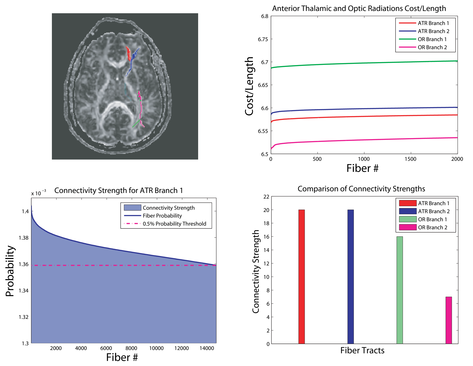

Figure 3.

References:

[1] Mori S, et al. 1999. Three-dimensional tracking of axonal projections in the brain by magnetic resonance imaging. Annals of Neurology. 45:265-269.
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[3] Wakana S, et al. 2004. Fiber tract-based atlas of human white matter anatomy. Radiology. 230:77-87.

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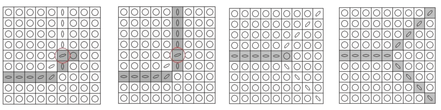

Figure 1.

Dynamic Programming

On the left, we define a probability law for labeling white matter tracts based on a sequentially additive cost function derived from the diffusion tensor viewed as a covariance operator in a Gaussian law. The probability law is associated with the quadratic form representation of the direction vectors of the diffusion tensor matrix between neighboring nodes along paths through the brain (Eq. 1). Generation of a maximum probability path is equivalent to minimizing a sequentially additive energy function associated with arcs linking nodes along the path (Eq. 2). Dynamic Programming is employed to determine the optimal path between start and end regions by efficiently searching over all paths connecting the points, and choosing the one in which the total energy is minimized. This algorithm is then extended to track groups of paths in order to reconstruct white matter tracts in vivo [2].

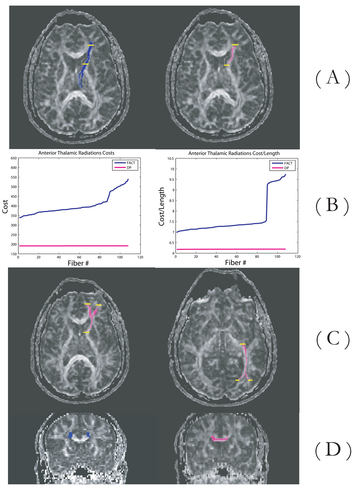

Figure 2.

Summary

Because axonal trajectories could previously be studied only by invasive in vivo studies, the utility of a non-invasive axonal reconstruction technique is significant. The primary advantage of dynamic programming is that it provides an explicit probabilistic representation of fiber paths, allowing the algorithm to track the optimal path between regions. Since this technique does not rely on following the principal eigenvector, tracking paths that are disrupted by noise, contain branching points, or end in a region of low anisotropy is possible. Furthermore, this statistical approach allows for the computation of a connectivity strength metric based on the chosen probability threshold that can be used to compare the connection strength of fiber tracts within and between subjects. By providing a quantitative measure of connectivity strength between regions in the brain, this method has the potential to be used in future longitudinal, population, and neuropathological studies.

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