

# Metrics for Computational Anatomy and Image Analysis

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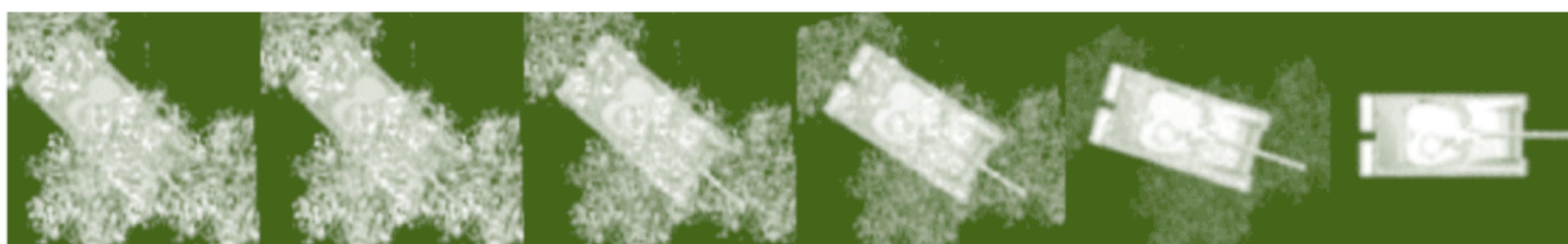
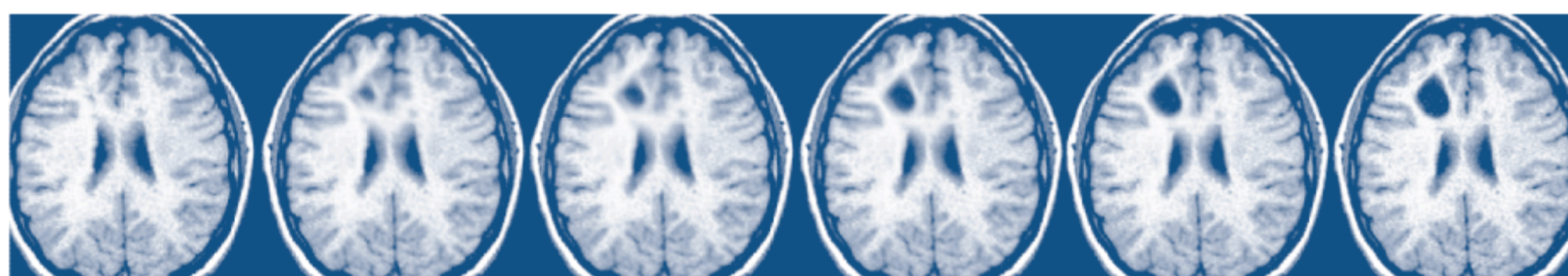
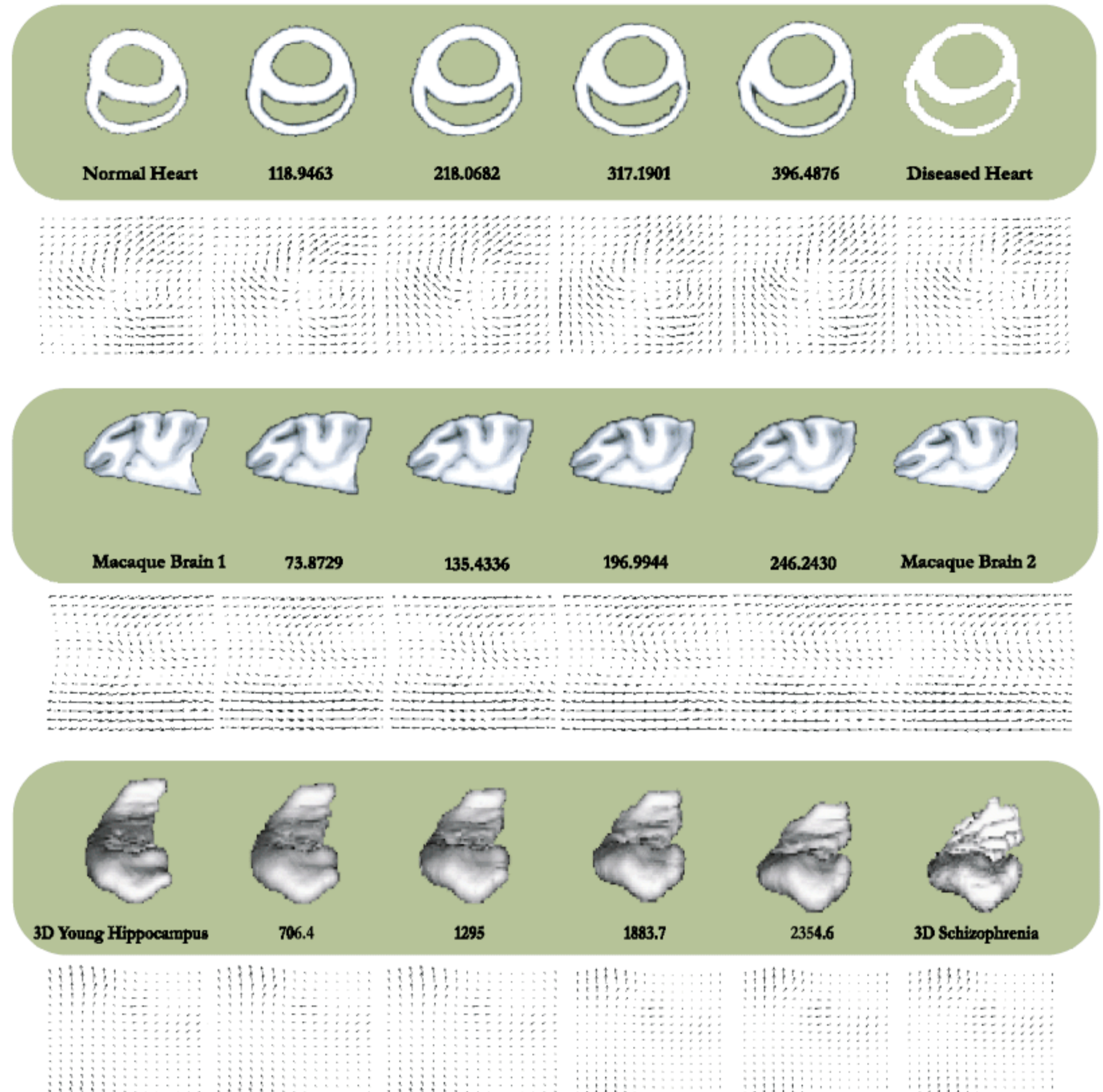
## Metrics for Homogenous Image Space

One of the Holy Grails of Image Analysis is the construction of metric space structure for families of images. In the emerging discipline of Computational Anatomy, this amounts to the placement of all the observed anatomical images in metric spaces, akin to the metric space structure which forms the basis of modern communications engineering. Unlike the  $L^2$  spaces of modern communications, this metric space is a curved Riemannian manifold, with the metric length between elements computed via the shortest path variational formulation. The metric space structure allows the precise quantification of the shape and size of objects represented in the images.

The anatomical imaging model is the deformable template model, where the set of images  $I$  is an orbit under the group of diffeomorphisms  $\mathcal{G}$  of the underlying coordinate space. The set  $I$  is a homogenous space, with all images in the set being topologically equivalent. Given any two images  $I_0, I_1$  in the orbit, there exists a diffeomorphism between them. The group of diffeomorphisms  $\mathcal{G}$  is an infinite-dimensional Riemannian manifold with the choice of a Riemannian metric in the tangent space of the manifold. It is a metric space with the metric distance between points as the length of the shortest path joining the points i.e. a geodesic on the manifold. The metric distance on the space of images is inherited from the metric structure on the space of diffeomorphisms. Given any two images, and the diffeomorphism  $\phi_1$  matching the images  $I_1 = I_0(\phi_1^{-1})$ , the metric distance between the images is:

$$\rho(I_0, I_1) = \inf_{\substack{\phi(\cdot, t) = v(\phi(\cdot, t), t): \\ \phi(\cdot, 0) = id, I_1 = \phi(\cdot, 1)^{-1} I_0}} \sqrt{\int_0^1 \|v(\cdot, t)\|_L^2 dt}$$

Shown to the right are the result of computing metric distances between images  $I_0$  on the far left and  $I_1$  on the far right taken from normal and diseased canine heart, Macaque brain cortex and 3D hippocampus volumes of an individual with Schizophrenia and a Young control. The row of images show a sample of the sequence on the geodesic corresponding to the transformation of the image  $I_0$  to the image  $I_1$ . The row of vector fields below show the representative sample of vector fields that generate the diffeomorphism and the numbers are the corresponding metric distance from the image  $I_0$ .



## Metrics for Neoplasm and Signature Variability

Image analysis must deal with *neoplasm* i.e. presence of new structures such as a tumor, the presence of highly variable clutter across images or variability due to lighting conditions and texture variation etc. as shown in the images on the left. The metric space structure must be extended to such non-homogenous image spaces.

The metrics formulated for this setting allow, in addition to a geometric transformation of the coordinate space, the image values to change from one image to another as a function of time modelled as  $J(y, t) = I(y, t)$ , allowing for creation or destruction of structures in an image where the corresponding structures do not exist in the other. This corresponds to construction of the metric in the product space  $\mathcal{X}$  with the metric distance between points  $I$  and  $I_1$  to be:

$$\rho(I_0, I_1) = \inf_{v, J: J(\cdot, 0) = I_0, J(\cdot, 1) = I_1} \sqrt{\int_0^1 \|v(\cdot, t)\|_L^2 + \left\| \frac{d}{dt} J(\cdot, t) + v(\cdot, t)' \nabla J(\cdot, t) \right\|^2 dt}$$

### References:

- U. Grenander and M. I. Miller, "Computational Anatomy: An emerging discipline," Quarterly of Applied Mathematics, vol. 56, pp. 617-694, 1998.
- M. I. Miller and L. Younes, "Group Actions, homeomorphisms and matching: a general framework," International Journal of Computer Vision, vol. 41, pp. 61-84, 1999.

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