Smooth Functional and Anatomical Maps via Cortical Harmonics of the Laplace-Beltrami Operator

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Introduction

Maps of curvature, cortical thickness, and functional data, indexed over the local coordinates of the cortical manifold play an important role in psychiatric neuroimaging studies. Due to the highly convoluted cerebral cortex, these maps are generally uninterpretable without proper methods of association and smoothness onto the local coordinate system. We generalize the spline smoothing problem for the sphere to the cortical surface by first computing numerical solutions to cortical harmonics of the Laplace-Beltrami (LB) operator with Neumann boundary conditions. Then the smoothed representations of functional or anatomical maps are described as a linear combination of cortical harmonics.

Laplace-Beltrami Cortical Harmonics

The eigenvalue problem of the LB operator for the cortical manifold is posed as

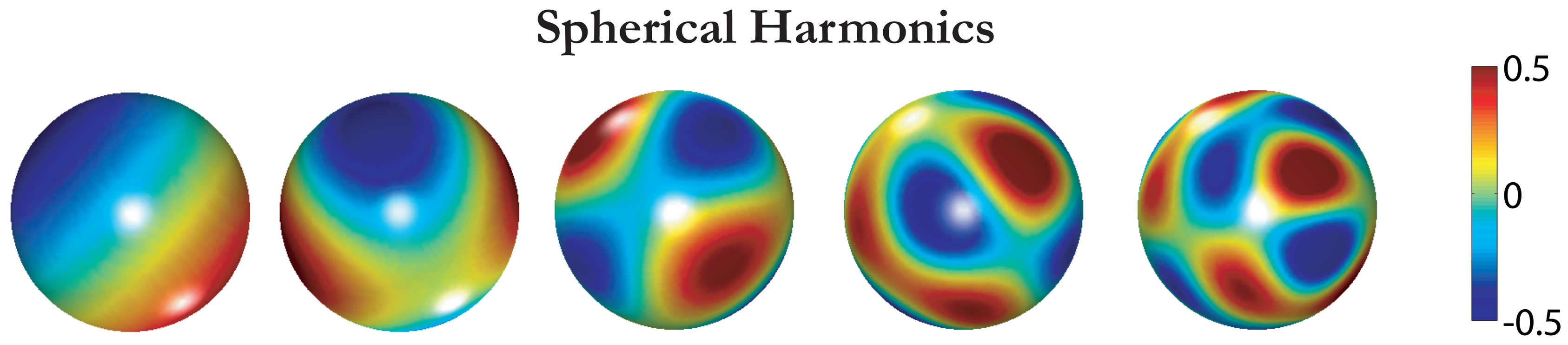
$$\Delta\varphi(u)+\lambda\varphi(u)=0 \text{ in } M, \quad \int_M |\varphi(u)|^2 dM = 1, \quad \left\langle \nabla\varphi(u),\vec{n}\right\rangle\Big|_{\partial M}=0,$$

with the corresponding weak form

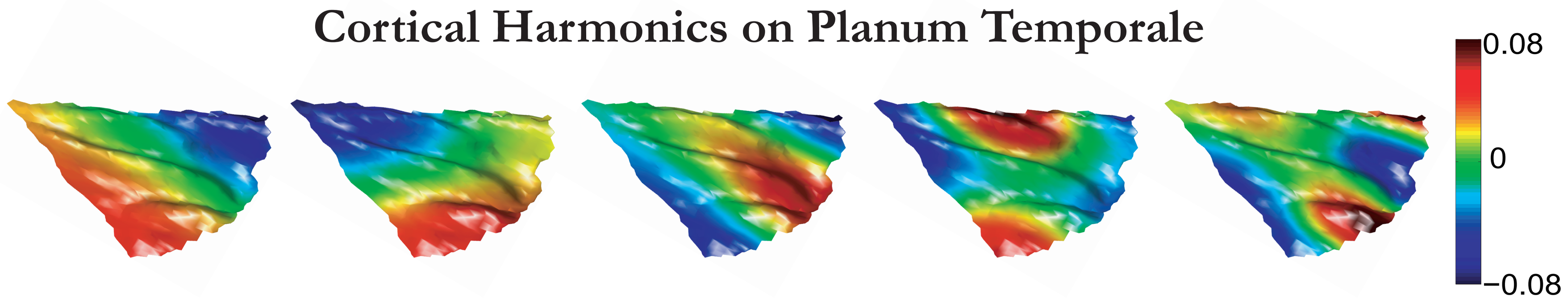
$$E=\int_M \|\nabla\varphi(u)\|^2 dM - \lambda\int_M |\varphi(u)|^2 dM.$$

$\{\varphi(\cdot),\lambda\}$ are critical points of E which are found via finite element methods.

Spherical Harmonics

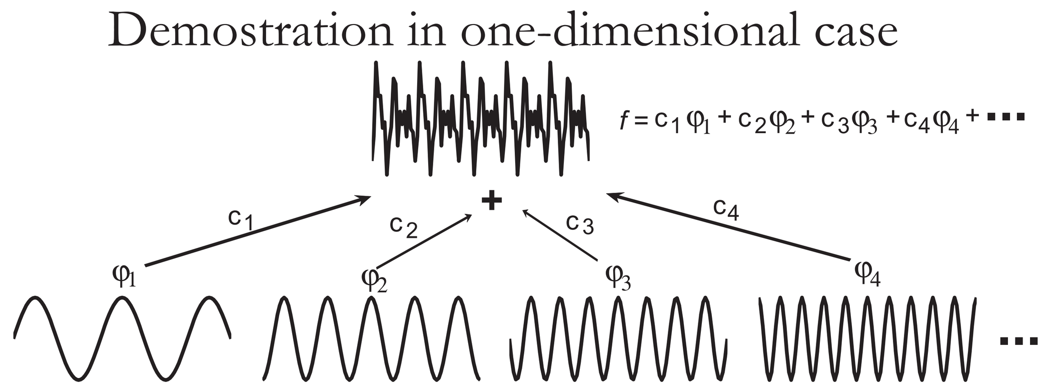


Cortical Harmonics on Planum Temporale

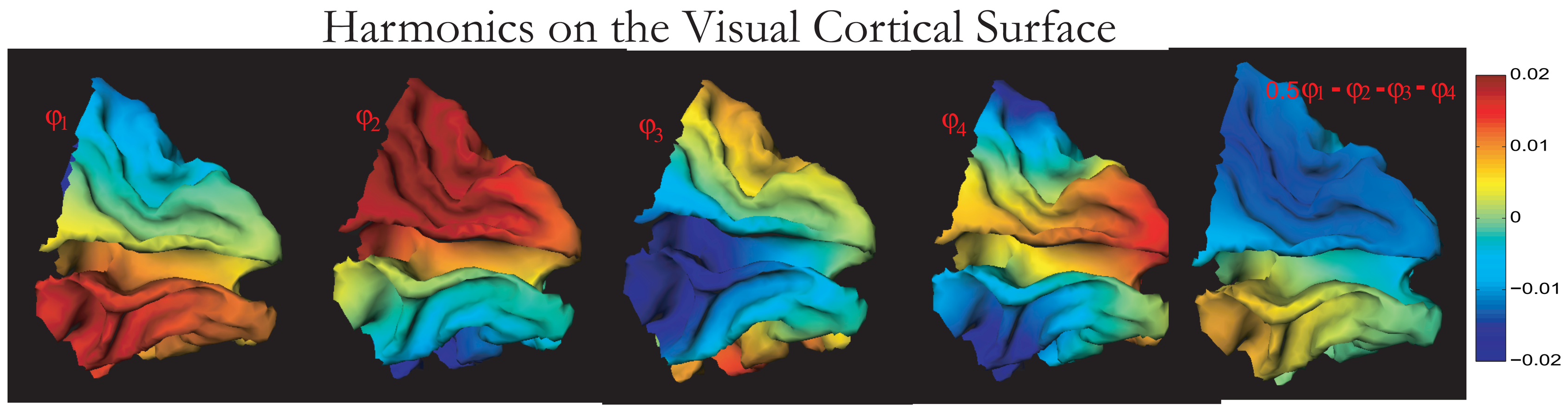


Spline Smoothing

Demostration in one-dimensional case



Harmonics on the Visual Cortical Surface



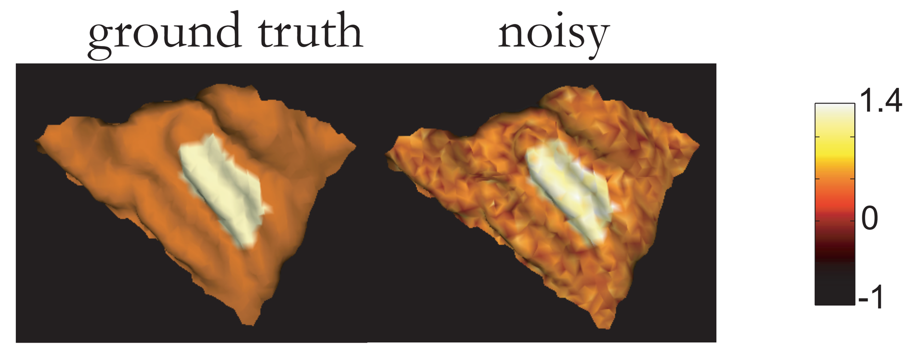
The spline smoothing problem is posed as

$$f(\cdot)=\arg\min_{f(\cdot)} \gamma\int_M \|\nabla f\|^2 dM + \sum_{i=1}^{N}(f(p_i)-y_i)^2,$$

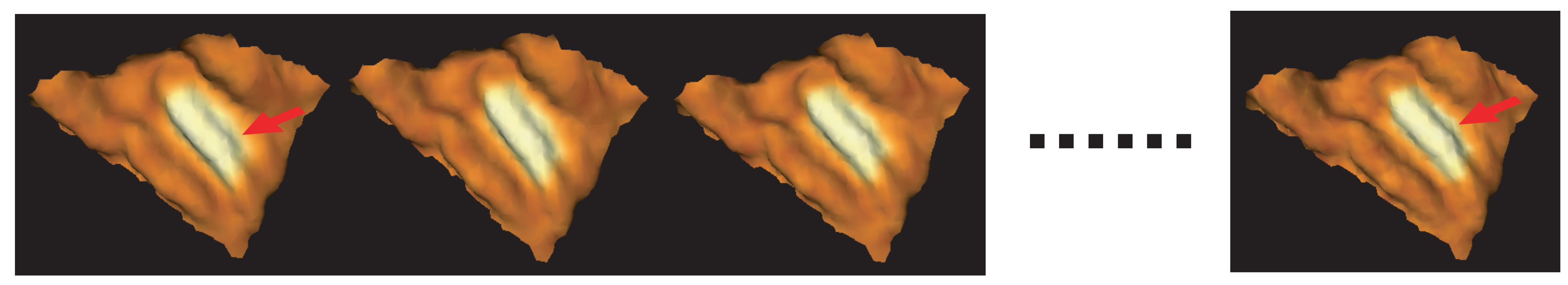
giving the minimizer $f(\cdot)=f_0+\sum_{k=1}^{N}\beta_k G(\cdot,u_k)$, where the reproducing kernel is $G(u_1,u_2)=\sum_{i=1}^{\infty}\frac{1}{\lambda_i}\varphi_i(u_1)\varphi_i(u_2)$. The solution $\hat{f}_0, \hat{\beta}$ satisfy the linear system :

$(G+\gamma I)\hat{\beta}+\hat{f}_0\mathbf{e}=Y$ and $\mathbf{e}^T\hat{\beta}=0$.

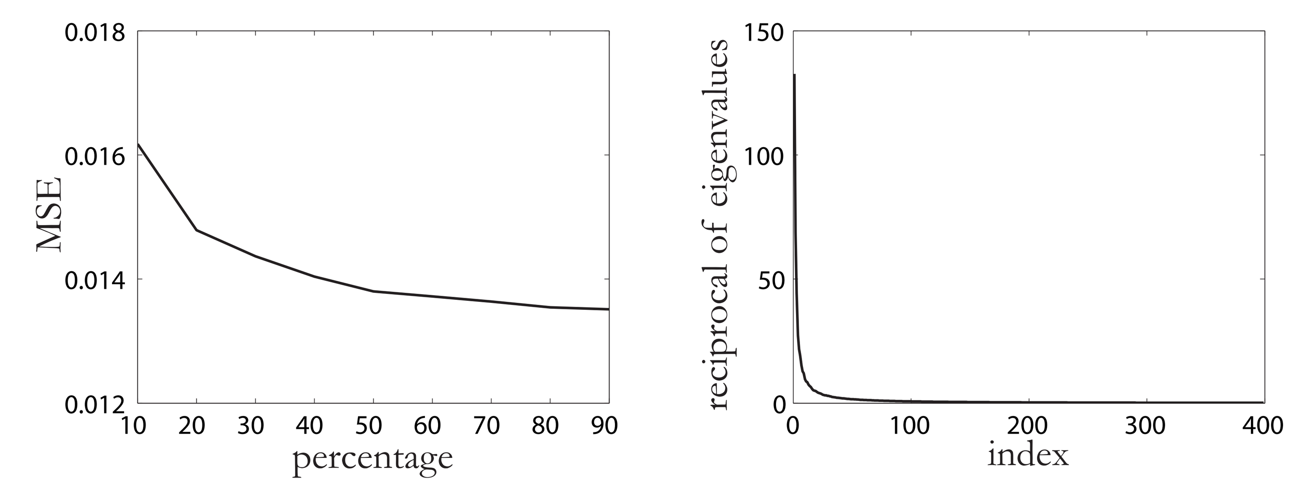
Quantative Evaluation



smoothed results

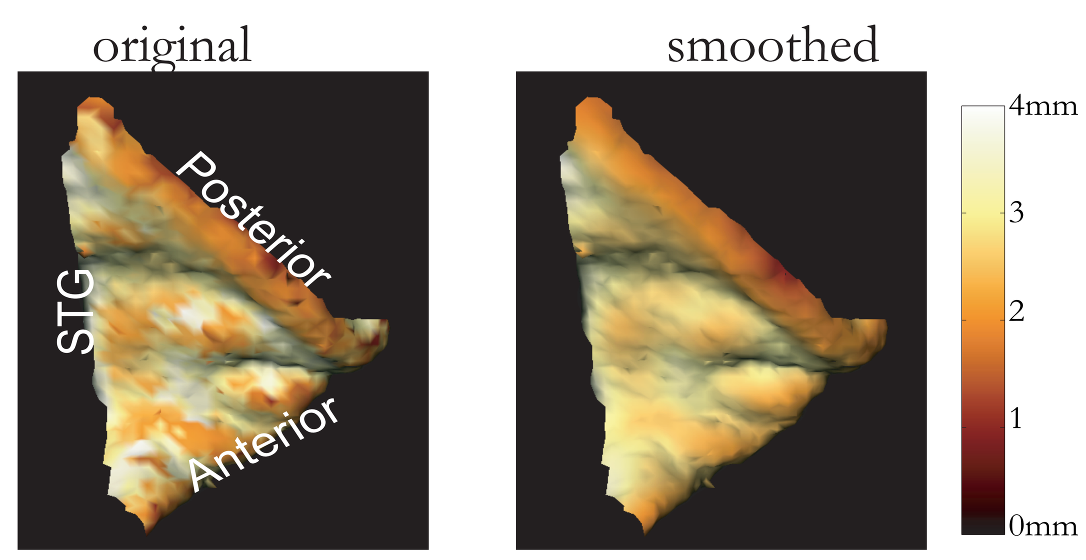


10%, 20%, 30%, ... , 90% of total number of harmonics were used in the smoothing.

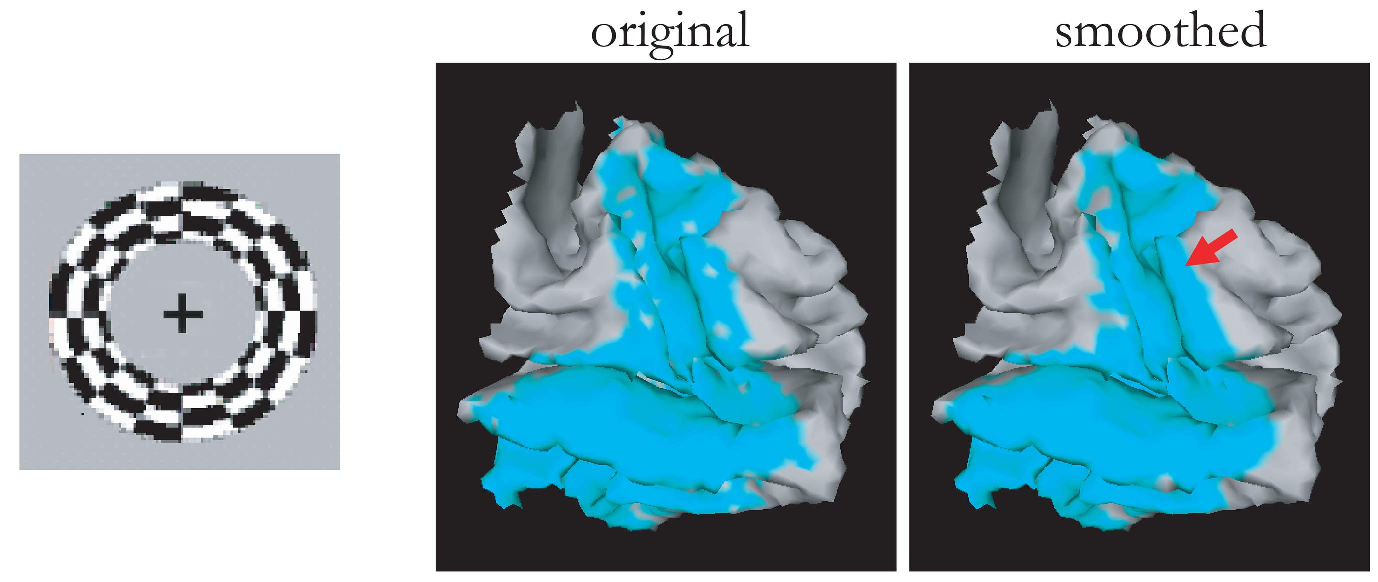


Above left figure shows the mean square error (MSE) between the smoothed results and the ground truth as a function of percentage of harmonics involved. The small change in MSE implies subtle difference among these smoothed results. This is partly because the reciprocal of eigenfunctions decreases rapidly (see right figure).

Cortical Thickness Map



Functional Response Map



Conclusion

We have generalized the spline smoothing problem for a unit sphere to an arbitrary Riemannian manifold with boundaries. The synthetic example suggests that a small number of harmonics are needed for the smoothing. Applications in smoothing cortical thickness and functional statistical maps on submanifolds of the neocortex will be useful for statistical analysis.

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